Variance computation for system matrices and transfer function from I/O subspace system identification.

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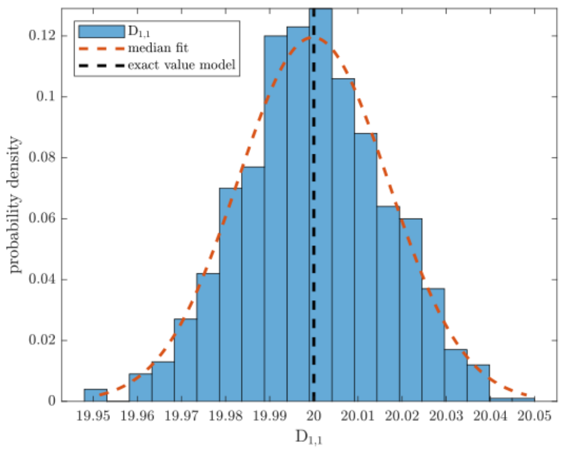
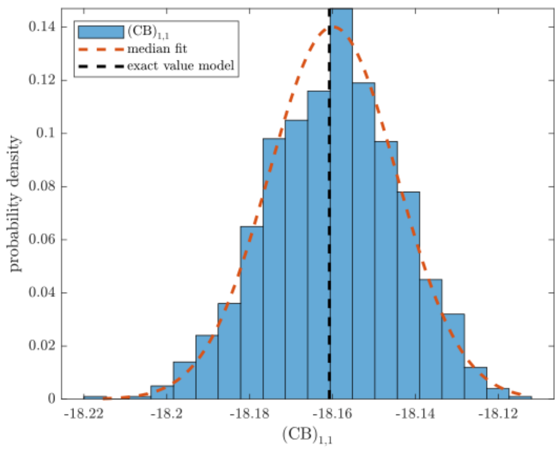
Contributions:

- Simple estimation of **B** and **D** in input/output stochastic system identification,
- Simple and explicit expression for the covariance estimation of **B** and **D**,
- Simple an explicit expression for the covariance estimation of **G(s)**.



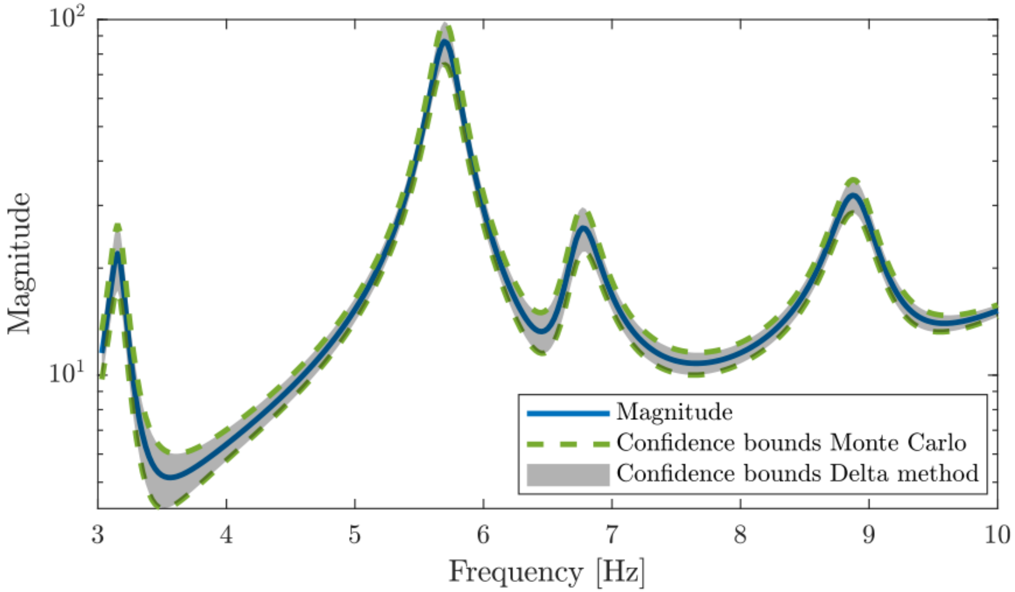


Algorithm:

- Propagation of covariance of data covariance sequences to estimates of **B** and **D**,
- Covariance propagation of estimates of **A**, **B**, **C**, **D** to estimate of **G(s)**.

Result:

- Confidence intervals of **B**, **D** and **G(s)** components.



Background

Dynamic behavior of an LTI system without feedback is fully described by the discrete-time state space model

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + Du_k + v_k \end{aligned}$$

Assumptions:

- w_k and v_k are white noise processes,
- u_k is persistently exciting input

The quadruplet **A**, **B**, **C**, **D** can be estimated from *k = 1…N* input/output data samples with using e.g. *N4SID* algorithm.

Statistical properties of estimates of **A**, **B**, **C**, **D**:

- Asymptotically Gaussian distributed, under persistently exciting white noise input,
- Expressions for the asymptotic covariance of **A**, **B**, **C**, **D** estimates exist, but require Q, R, S noise covariances to be computed

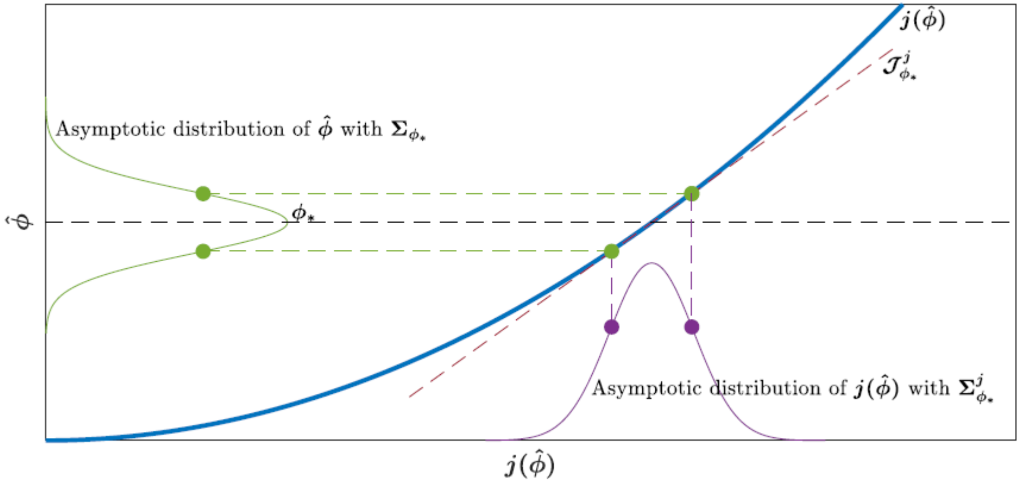
Objective

Derive simple expressions for the covariance related to A,B,C,D estimates e.g., relating their covariance directly to the (sample) covariance of I/O data

A. Chiuso and G. Picci. The asymptotic variance of subspace estimates. *Journal of Econometrics*, 118(1):257 – 291, 2004.

D. Bauer and M. Jansson. Analysis of the asymptotic properties of the MOESP type of subspace algorithms. *Automatica*, 36(4):497 – 509, 2000.

Statistical Delta method for variance propagation



CLT for $\hat{\phi}$: $\sqrt{N}(\hat{\phi} - \phi_*) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\phi_*})$

Small perturbation in $j(\hat{\phi})$: $\Delta j(\hat{\phi}) = j(\hat{\phi}) - j(\phi_*) \approx \mathcal{J}^j_{\phi_*} \Delta\hat{\phi}$

CLT for $j(\hat{\phi})$: $\sqrt{N}\left(j(\hat{\phi}) - j(\phi_*)\right) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma^j_{\phi_*})$, where $\Sigma^j_{\phi_*} = \mathcal{J}^j_{\phi_*} \Sigma_{\phi_*} (\mathcal{J}^j_{\phi_*})^T$

generic covariance expression

N4SID: $\mathcal{H} = \mathcal{Y}^+ /_{\mathcal{U}^+} \mathcal{W}^-$, where $\mathcal{Y}$, $\mathcal{U}$, $\mathcal{W}$ are data matrices

'Squared' Hankel matrix: $\mathcal{H}^s = \mathcal{H}\mathcal{H}^T$

Small perturbation in $\mathcal{H}^s$: $\text{vec}(\Delta\mathcal{H}^s) = \mathcal{J}^{\mathcal{H}^s}_{\mathcal{R}} \text{vec}(\Delta\mathcal{R})$

where $\mathcal{R}$ is data covariance

Small perturbation in (A,C) :

$$\text{vec}(\Delta A) = \mathcal{J}^A_{\mathcal{R}} \text{vec}(\Delta\mathcal{R})$$

$$\text{vec}(\Delta C) = \mathcal{J}^C_{\mathcal{R}} \text{vec}(\Delta\mathcal{R})$$

Simple to obtain from data

Novelty

$$\text{vec}\left(\Delta \begin{bmatrix} D \\ B \end{bmatrix}\right) = \mathcal{J}^{D,B}_{\mathcal{R}} \text{vec}(\Delta\mathcal{R})$$

$$\text{vec}(\Delta G(s)) = \mathcal{J}^{G(s)}_{A,B,C,D} \mathcal{J}^{A,B,C,D}_{\mathcal{R}} \text{vec}(\Delta\mathcal{R})$$

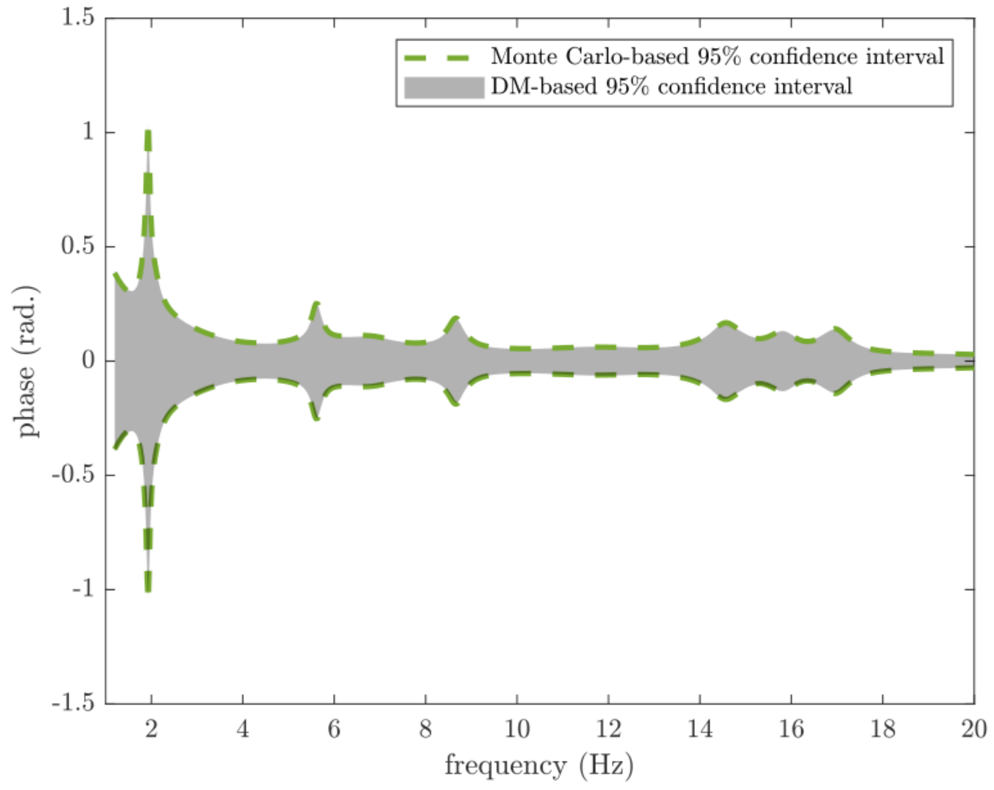
G. Casella and R. L. Berger. *Statistical Inference*. Cengage Learning, 2nd edition, 2001.

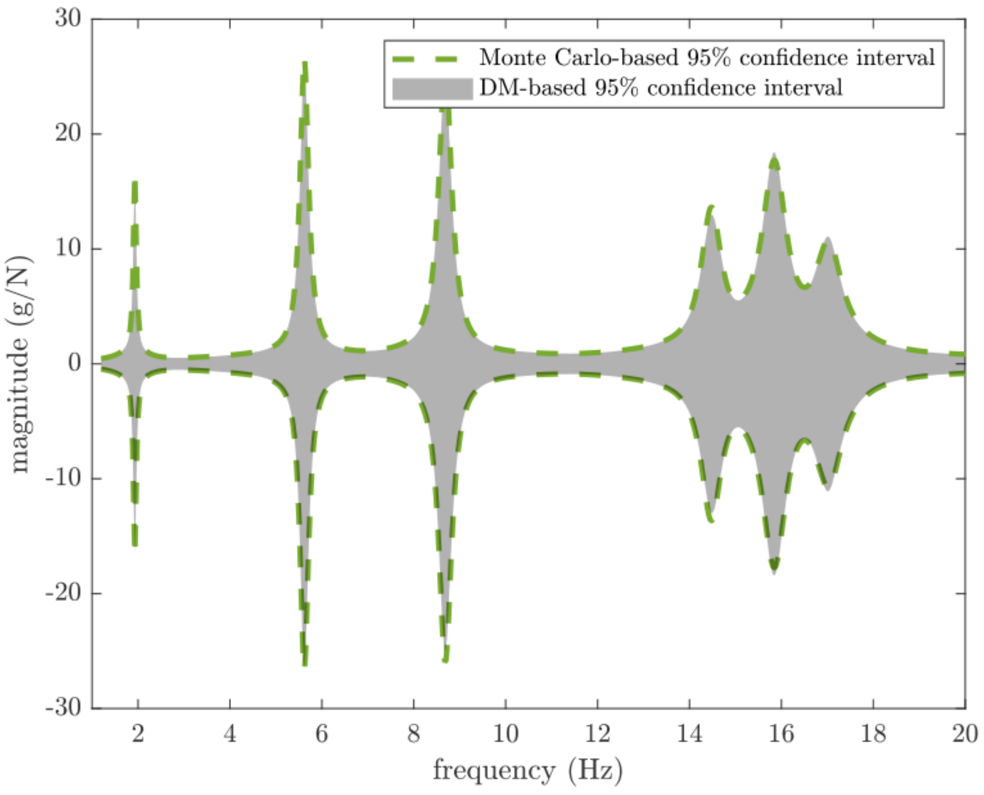
P. Mellinger, M. Döhler, and L. Mevel. Variance estimation of modal parameters from output-only and input/output subspace-based system identification. *Journal of Sound and Vibration*, 379(C):1 – 27, 2016.

M. Döhler and L. Mevel. Efficient multi-order uncertainty computation for stochastic subspace identification. *Mechanical Systems and Signal Processing*, 38(2):346–366, 2013.

Numerical results

Validation of the confidence intervals based on a chain system simulation

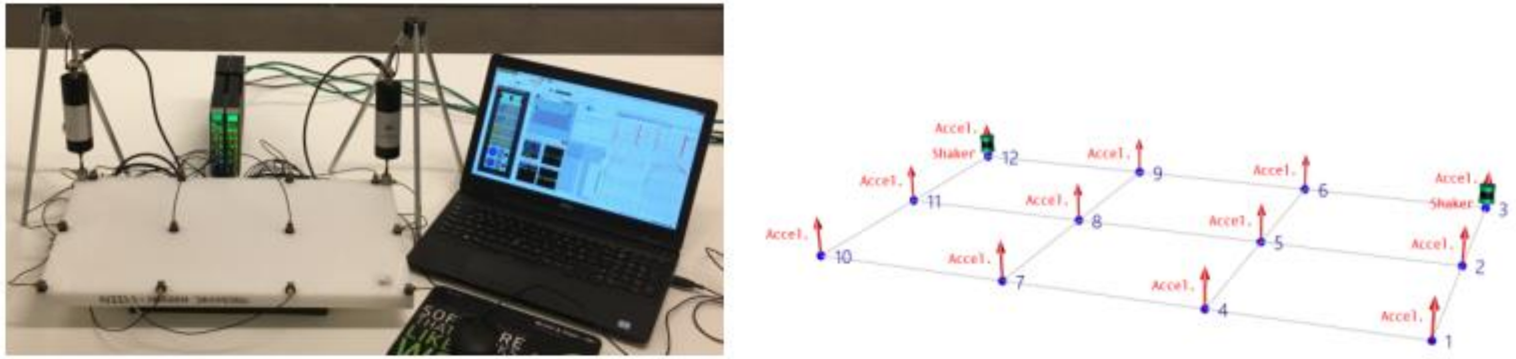




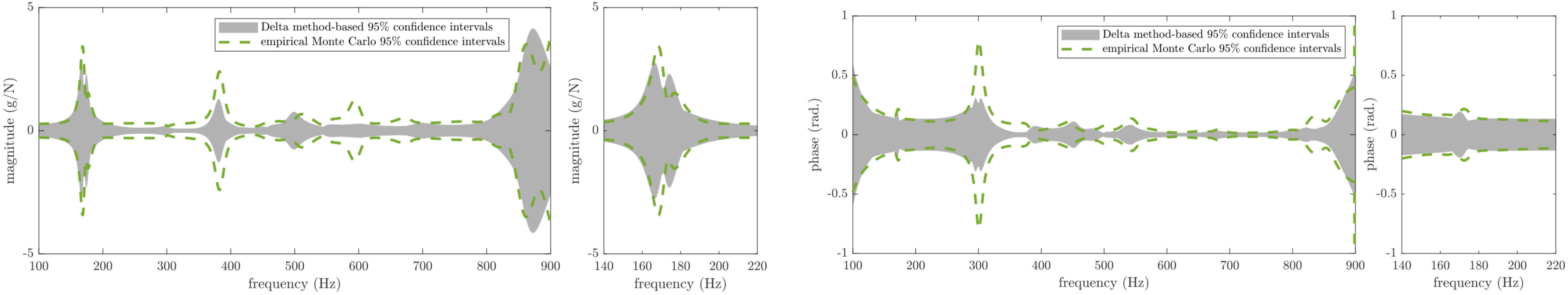
Zoomed x20 Monte Carlo and delta method-based 95% confidence intervals of **one component** the phase and the magnitude of G(s)

Real-data results

Experimental setup:
- rectangular plate of high-density plastic,
- 2 white noise inputs from shakers,
- 12 acceleration outputs,
- fs=8192 Hz and N approx. 1,000,000 samples.



Experimental plate setup



Experimental Monte Carlo and delta method-based 95% confidence intervals of the phase and the magnitude of G(s)