A layer potential approach to functional and clinical brain imaging

Masimba Nemaire[1,2], Paul Asensio[1], Jean-Michel Badier[3] Juliette Leblond[1], Jean-Paul Marmorat[4]

[1]FACTAS, Inria Sophia Antipolis, France, [2]Institut de Mathématiques de Bordeaux, France
[3]Institut de Neurosciences des Systémes, France, [4]Center of Applied Mathematics, France

Introduction

In functional and clinical brain imaging, the aim is to recover the location of brain activity associated with the performance of a task and events such as epileptic seizures, respectively. This is achieved by solving inverse problems using electric and magnetic data that is collected in non-invasive manners such as in EEG and MEG (for electric and magnetic data, respectively) or in an invasive manner such as in sEEG for electric data. In recent years models for the electro-magnetic fields were based the BEM symmetric methods. These methods require the use of the four boundary integral operators that include the double and single layer potentials. Here we propose a method that uses only the double and single layer potentials in combinations the method of fundamental solutions. Instead of modelling primary currents as dipolar sources we model them as $(L^2)^3$ vector-fields that result in distributed solutions in the source localisation problems. The method we introduce in this work is computationally less complex than the symmetric methods based approaches and unifies solution to functional and clinical brain imaging problems.

1. Functional and clinical brain imaging: Layer potentials

Let $\Omega \subset \mathbb{R}^3$, be a bounded Lipschitz domain with boundary, $\partial\Omega$. We model sources (primary currents) by vector-fields $\mathbf{M} \in [L^2(\partial\Omega)]^3$, which are normally oriented to $\partial\Omega$, hence $\mathbf{M} = M\nu$, where ν is the outward pointing unit normal to $\partial\Omega$. We have that the double and single layer potentials associated with $\mathbf{M}$ are respectively given by

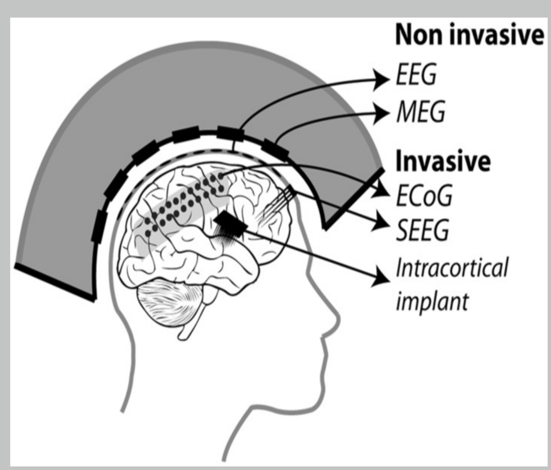
$$KM(x) = \frac{1}{4\pi}\int_{\partial\Omega} M(y)\frac{(x-y)}{|x-y|^3}\cdot\nu(y)\,d\mathcal{H}(y), \quad x \in \mathbb{R}^n\backslash\partial\Omega,$$

$$SM(x) = \frac{1}{4\pi}\int_{\partial\Omega} M(y)\frac{1}{|x-y|}\,d\mathcal{H}(y), \quad x \in \mathbb{R}^n\backslash\partial\Omega,$$

where $\mathcal{H}$ is the 2-dimensional Hausdorff measure on the surface $\partial\Omega$. When $x \in \partial\Omega$ the integrals above are taken in the principal value sense and are well defined. These two layer potentials form the basis of the expressions of the electromagnetic fields generated by $\mathbf{M}$.

3. Functional and clinical brain imaging: Data acquisition

The figure on the right shows the placement of the different sensor types for data acquisition used in functional and clinical brain imaging.
EEG, SEEG, ECoG and intra-cortical micro- electrodes measure v,
SEEG has a higher signal to noise ratio than EEG.
MEG sensors measure a component of $\mathbf{B}$, this makes MEG more sensitive to "tangential" sources than "radial" sources which is to say that it is more sensitive to sources on sulci than on gyri of the brain.
MEG offers better spatial resolution than EEG because magnetic fields are less distorted by the skull and scalp.
Further SEEG has a higher spatial resolution than EEG as EEG suffers from the attenuating effect of the skull's low electrical conductivity.



FM for v is valid at arbitrary points in space hence it can be used for the forward problems of SEEG and EEG. **FM** for $\mathbf{B}$ also works at arbitrary points on space hence we can use in the forward problem for MEG.
In the recovery problem we will use MEG, EEG and SEEG data.

4. Functional and clinical brain imaging: Source recovery problem

We wish to recover a normal vector-field (primary current) of the form $M\nu$ where M is an L^2 function on the grey-white matter interface such that, if we apply **FM** to the function, it reproduces the given data. We will however solve a discrete version of this **inverse problem** (**IP**) for source recovery. We triangulate the surfaces that make up the head and we discretise M to be a linear combination of linear basis functions on triangles. The gist of the approach is: this discretised model, which is a first order approximation of the continuous problem, has the remarkable property that **FM** can be computed exactly, see [2]. Hence we are left to recover the coefficients of the linear expansion of M into basis functions. This **IP** is ill-posed because of the non-uniquness of solutions, as we only have point-wise data. The solution is non-unique since given a solution, f of the **IP**, we can find a f_0 such that $\mathbf{FM}(f+f_0)$=$\mathbf{FM}(f)$ at the points where data is acquired. Hence we solve the following regularised problem, which is well-posed: Given discrete SEEG, EEG and MEG data contaminated with noise, $\mathbf{d}$, find $\mathbf{u}^*$ such that

$$\mathbf{u}^* = \arg\min_{\mathbf{u}} \|\mathbf{FM}(\mathbf{u}) - \mathbf{d}\|_2^2 + \lambda\,\|\mathbf{Ru}\|_2^2,$$

where **FM** is the discretised forward model, $\lambda \in \mathbb{R}_+$ and $\mathbf{R}$ is a regularisation matrix . The solution to the above problem is given by

$$\mathbf{u}^* = (\mathbf{FM}^T\mathbf{FM} + \lambda\mathbf{R}^T\mathbf{R})^{-1}\mathbf{FM}^T\mathbf{d}.$$

Once the coefficients $\mathbf{u}^*$ are known we use them with the linear basis to find the source. Note that the solution depends on λ and we have to choose the λ that gives the best result. We use the L-curve to choose the best λ. $\mathbf{R}$ can be chosen to be the identity matrix or such that $\|\mathbf{Ru}\|_2$ is the L^2-norm of the discretised function.

6. Conclusions

- The method presented here is computationally less complex than the existing symmetric methods and provides a unified approach to all functional and clinical brain imaging techniques, including ECoG and intracortical implants.
- The recovery technique we have introduced solves the source recovery problem by taking into account the regularity of the electric potential and/or the magnetic flux density with the head volume. This has two advantages of:
  - solving the cortical mapping problem for the electric potential,
  - providing additional data to work with in the recovery in order to make the problem less under-determined hence improving the resulting source recovery.

Acknowledgements

The work of the first author is supported by the Agence Nationale de la Recherche (ANR) through the REPKA project.

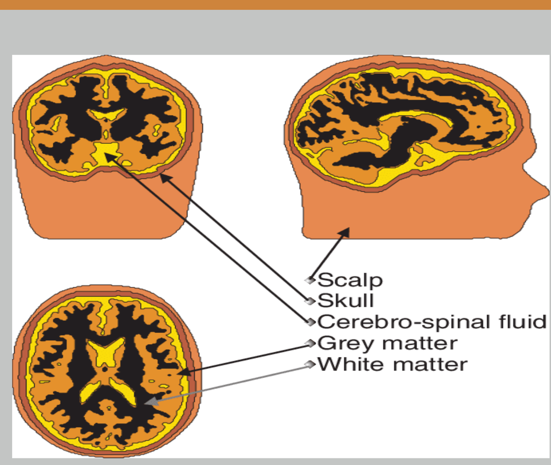
References

[1] P. Asensio, L. Leblond, J.-P. Mamorat, M. Nemaire. A layer potential approach to brain imaging, in preparation.

[2] S. Nintcheu Fata. Explicit expressions for 3D boundary integrals in potential theory, International Journal for Numerical Methods in Engineering, 78, 32-47, 2009.

[3] J. Sarvas. Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem. Physics in medicine and biology, 32 1:11-22, 1987.

2. Functional and clinical brain imaging: Forward Model (FM)



For brain imaging we consider a head model made of three concentric layers each of which has a different (constant) electrical conductivity and uniform magnetic permeability. These layers represent the **brain**, the **skull** and the **scalp**. The brain is further partitioned into two layers, the **white matter** and the **grey matter**, the latter being the outer layer of the brain volume whose outer most surface is called the **cortical surface**, see **figure on the left**. Neurophysiological considerations suggest that the electromagnetic activity of the brain originates on the **grey-white matter interface**.
We model these sources by L^2 vector-fields normally oriented to the grey-white matter interface. Such vector-fields represent the primary current in the model of brain activity and they model the primary electrical and vector magnetic potentials using double and single layer potentials, respectively, see [1]. This is the first step in building the **forward model** (**FM**) for the electrical potential, v, and magnetic flux density, **B**; **FM is used to compute the electromagnetic activity of the brain for a given a source**.
v and $\mathbf{B}$ satisfy the following differential equations

$$\sigma\Delta v = 0 \text{ in } \mathbb{R}^3,$$
$$\nabla\cdot\mathbf{B} = 0 \text{ in } \mathbb{R}^3.$$

It is known that v is continuous in the head volume, especially across the surfaces of discontinuity of the electrical conductivity while $\partial_n v$ is discontinuous across these surfaces, see [3], $\partial_n v$ is assumed to be zero on the scalp. This allows to build **FM** of v.
The magnetic property of a source we are interested is **B** (more specifically a given direction of **B**). **B** is the curl of the vector magnetic potential and also depends on v, hence **FM** of **B** for a given source depends on v generated by the same source. This gives us a way of coupling the electrical and magnetic data that can be obtained through EEG and MEG modalities.

5. Numerical results

We present some numerical results for the source recovery based on the recovery method just presented. The data was generated using OpenMEEG which is based on the symmetric method using a dipolar source model.

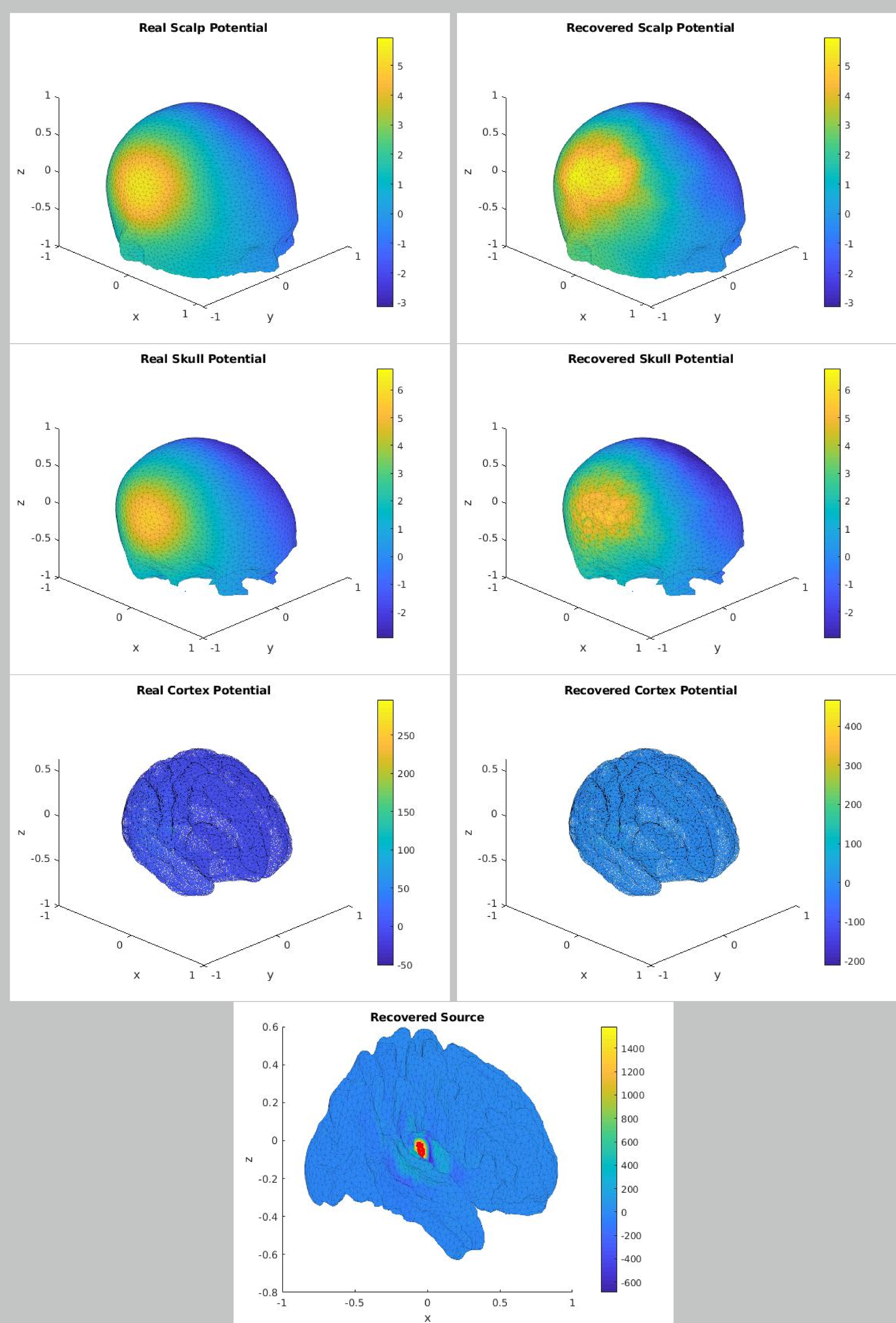


Figure: To localise the source $\mathbf{M} = M\cdot\nu$ we look at the support of the extremal values of M. In the bottom most figure above, the support of the extremal values of M is in yellow. The red dots represent the true locations of the dipolar source.

https://team.inria.fr/factas/ masimba.nemaire@inria.fr