Bayesian tensor network-based Volterra system identification

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September 2021

Delft University of Technology
ERNSI 2021

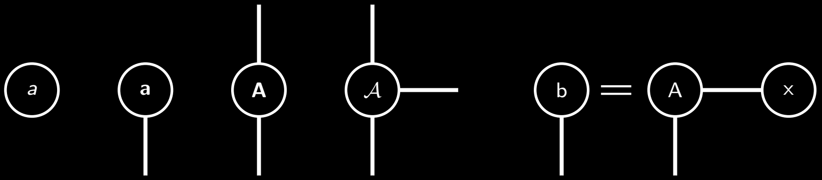
Problem Setting

- Identifying nonlinear MIMO Volterra system

$$y(t) = w_0 + \sum_{p=1}^{P} \sum_{m_1=0}^{M} \cdots \sum_{m_p=0}^{M} w_p(\tau_1, \ldots, \tau_p) \prod_{j=1}^{P} u(t - \tau_j)$$

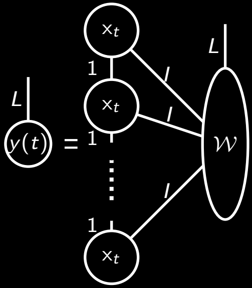
  for given $\mathcal{D} = \{u(t), y(t)\}_{t=1}^{N}$
- Curse of Dimensionality: D-th order Volterra kernel contains $(PM)^D$ elements
- No uncertainty quantification of Volterra weights for noisy data

Basic Graphical Notation

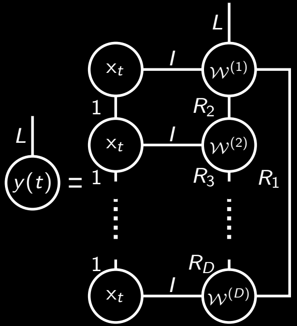


scalar vector matrix tensor matrix-vector equation $b = Ax$

Volterra Tensor Network (TN)

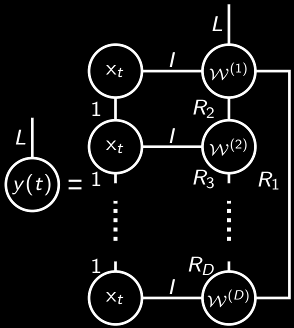


Volterra tensor $\mathcal{W}$ with $(PM+1)^D$ elements, $I = PM + 1$

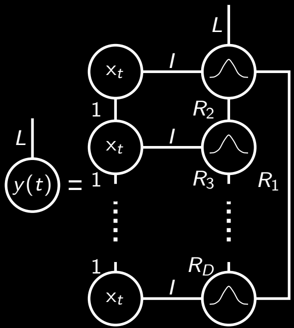


TN - cores $\mathcal{W}^{(d)}$ with $R_{d-1} \times (PM+1) \times R_d$ elements, $I = PM + 1$

Bayesian framework for Volterra Tensor Network (TN)



TN - cores $\mathcal{W}^{(d)}$ with $R_{d-1} \times (PM+1) \times R_d$ elements, $I = PM + 1$



TN-cores as probability density functions (PDF), $I = PM + 1$

Key Idea

Update TN-PDF with Bayes Theorem

$$P(\frown, \frown, \frown|\mathcal{D}) = \frac{P(\mathcal{D}|\frown, \frown, \frown)P(\frown, \frown, \frown)}{P(\mathcal{D})}$$

Summary

- express MIMO Volterra system as tensor network (TN)
- treat TN-cores as random variables expressed by probability density functions (PDF)
- update TN-PDF with Bayes Theorem
- obtain confidence bounds for model predictions
- avoid Curse of Dimensionality by TN-structure for Volterra PDF and covariance matrix

References

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