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AUTOMATIC CONTROL LABORATORY

Signal Matrix Model in Simulation, Signal Denoising and Control Design

Mingzhou Yin, Andrea Iannelli, Roy S. Smith
Automatic Control Laboratory, Swiss Federal Institute of Technology, 8092 Zurich, Switzerland

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1 Signal matrix model (SMM)

Why?

Conventional **system identification paradigms** rely on compact parametric models.

Challenges: systems are increasingly complex; how to use big data

Solution: moving from compact parametric models to implicit non-parametric trajectory models

Novelty: a statistically optimal approach to deal with noisy data

What?

Construct trajectory $\mathbf{z} = \mathrm{col}(\mathbf{u}, \mathbf{y})$ by combining **direct knowledge** and linear combination of **noise-corrupted signal matrix**.

Signal matrix: Hankel matrix of trajectory data

$$Z = \begin{bmatrix} z_0^d & z_1^d & \cdots & z_{M-1}^d \\ \vdots & \vdots & \ddots & \vdots \\ z_{L-1}^d & z_{L_0}^d & \cdots & z_{N-1}^d \end{bmatrix}$$

Preconditioning: compress by SVD

$$Z \overset{\text{svd}}{\longrightarrow} WSV^{\mathrm{T}}, \qquad \tilde{Z} \triangleq WS(:, 1{:}Ln_z)$$

Noise-free case: *Willems' fundamental lemma* (Willems, 2005)

known part → $\mathbf{z}_1 = Z_1 g$, unknown part → $\mathbf{z}_2 = Z_2 g^*(\mathbf{z}_1, Z_1)$

Noisy case: $\hat{\mathbf{z}}$ as trajectory measurements; g as hyper-parameters defining prior distribution of $\mathbf{z}$ by Z

$$\hat{\mathbf{z}} = \mathbf{z} + \mathbf{w}_z, \qquad \mathbf{w}_z \sim \mathcal{N}(0, \Sigma_z), \qquad \mathbf{z} \sim \mathcal{N}\left(Zg, \Sigma_{zg}(g)\right)$$

For unknown parts in $\hat{\mathbf{z}}$, corresponding elements in $\Sigma_z \to \infty$.

Empirical Bayes step: solve for g

$$\begin{aligned} g^\star &= \arg\max_g p(\hat{\mathbf{z}}|g) \\ &= \arg\min_g \operatorname{logdet}\left(\Sigma_{zg}(g) + \Sigma_z\right) + (\hat{\mathbf{z}} - Zg)^{\mathrm{T}}\left(\Sigma_{zg}(g) + \Sigma_z\right)^{-1}(\hat{\mathbf{z}} - Zg) \end{aligned}$$

MAP estimation step: solve for $\mathbf{z}$ given $g^\star$

$$\begin{aligned} \mathbf{z}^\star &= \arg\max_{\mathbf{z}} p(\hat{\mathbf{z}}|\mathbf{z}) \cdot p(\mathbf{z}) \\ &= \Sigma_{zg}(g^\star)\left(\Sigma_{zg}(g^\star) + \Sigma_z\right)^{-1}\hat{\mathbf{z}} + \Sigma_z\left(\Sigma_{zg}(g^\star) + \Sigma_z\right)^{-1} Z g^\star \end{aligned}$$

2 Applications

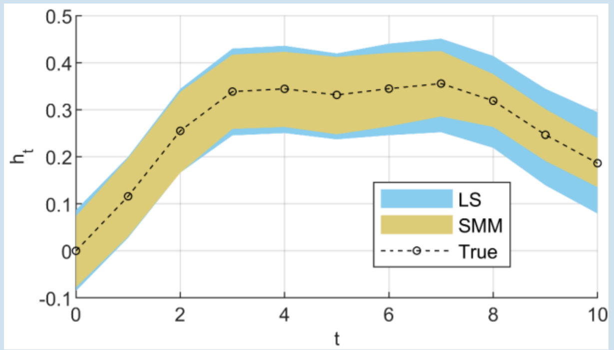
Simulation

Estimate outputs from known inputs and initial conditions.

Condition: $\mathbf{u}$ is known exactly, first outputs $(\mathbf{y}_i)_{i=0}^{L_0-1}$ are measured as initial condition

Prior knowledge of $(\mathbf{y}_i)_{i=L_0}^{L-1}$ can be added as Gaussian process.
e.g., stable spline kernels in impulse response simulation

Example: impulse response simulation **Benchmark:** least-squares estimation



Signal denoising

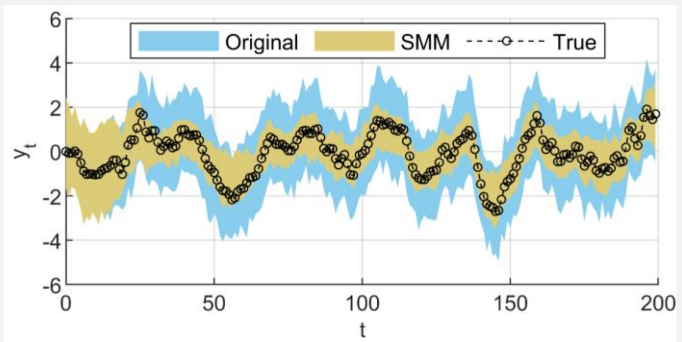
Denoise trajectory based on history trajectory data.

Condition: all the trajectories are measured with noise

Online data can be added to the signal matrix:

$$Z_{t+1} = \left[\gamma Z_t \quad (z_i)_{i=t-L+1}^{t}\right], \qquad \gamma\text{: forgetting factor}$$

Example: online signal denoising, Gaussian input



Control design

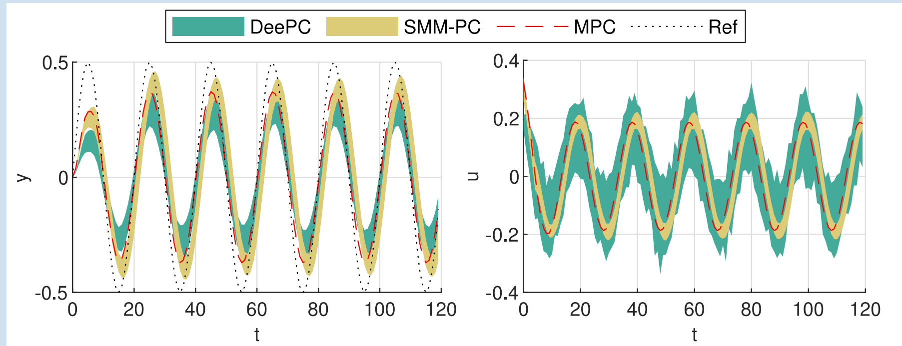
Optimal reference tracking by

$$\underset{\mathbf{u},\mathbf{y}}{\text{minimize}} \ \|\mathbf{y} - \mathbf{y}_{\text{ref}}\|_Q^2 + \|\mathbf{u} - \mathbf{u}_{\text{ref}}\|_R^2$$

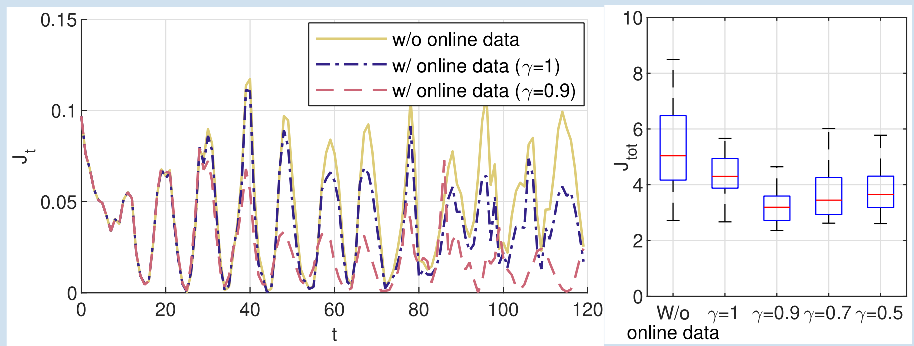
Condition: $(\boldsymbol{u}_i, \boldsymbol{y}_i)_{i=0}^{L_0-1}$ are measured past trajectory as initial condition; $(\hat{\boldsymbol{u}}_i, \hat{\boldsymbol{y}}_i)_{i=L_0}^{L-1}$ are set to reference trajectory, corresponding elements in Σ_z are proportional to Q^{-1} & R^{-1}.

Example: receding horizon, sinusoidal reference, no I/O constraints

Benchmark: ideal MPC & DeePC (*Coulson, 2019*)



Closed-loop trajectory comparison with ideal MPC & DeePC



Online data adaptation for system with slow parameter drifts

References

Mingzhou Yin, Andrea Iannelli, and Roy S. Smith. Maximum likelihood estimation in data-driven modeling and control. arXiv:2011.00925, 2020.

Mingzhou Yin, Andrea Iannelli, and Roy S. Smith. Maximum likelihood signal matrix model for data-driven predictive control. Proceedings of the 3rd Conference on Learning for Dynamics and Control, PMLR 144:1004-1014. arXiv:2012.04678, 2021.